Application of Linear Programming to Forest Resource Planning

By Howard A. Leach

1. What is Linear Programming?

Linear programming (LP) is a mathematical technique whose original and still primary application is in the petroleum industry: to calculate the optimum way to allocate each type of crude oil from each source to each refinery, and how much of each product should be made at each refinery for shipment to each market area. Because of the large increases in profit which result from making correct allocations, millions of dollars have been spent on refining the technique, and developing the computer programs required to carry out the calculations.

There is an obvious parallel in the forest industry: for crude oil, substitute timber; and for refinery, substitute sawmill, veneer or plywood plant, or pulpmill.

The procedure is to set up a mathematical model of the process, consisting of a set of simultaneous equations of a type that most people learn about in high-school algebra classes, and then promptly forget. The variables are the flows of materials or products from one place or process to another, and the equations are the relationships between these variables. Linear programming finds the values of these variables that will maximize the profitability of the process. The set of simultaneous equations is referred to as the "matrix", and matrix algebra is used to obtain the solution.

In high-school, it is usually taught that to obtain a solution there must be the same number of equations as unknowns. In real-life applications there are usually many more unknowns than equations, and hence many possible solutions. Also, solutions that yield negative values for the variables are not allowed (since you obviously can't have a solution that processes a negative amount of raw material). The problem is to find a solution that gives positive values for all the variables and ***maximizes the profit***, subject to constraints such as the capacities of the processing units.

Linear programming is not applicable to all types of process. For instance, many processes are inherently non-linear, in that outputs are not proportional to inputs. For some others, the optimum values of the variables that linear programming would optimize is obvious, and thus a linear programming model is not necessary.

Linear programming has been applied many times in the forest industry to optimize log allocation, and particulary to optimize the operation of plywood plants. In many cases, increased profits in the millions of dollars have resulted from its successful application.

Many analysts, including the writer, like the technique because, once learned, it is easy to apply and generates "a lot of bang for the buck". The problem, from a consultant's point of view, is that it is sometimes difficult to sell. Before the consultant can sell the idea of implementing a linear programming model, it is first necessary that management have recognized that the operation is made of many inter-related processes, that decisions in one area of the operation have impacts on other areas, and that significant changes in profitability can result from changes made in the process.

Experience shows that when properly applied, a linear programming exercise always shows that operations with any degree of complexity can be run more profitably if raw materials or products are allocated to manufacturing facilities more optimally. There are simply too many variables in most manufacturing operations for a human being to figure out the most profitable way to operate.

Managers who have used linear programming successfully, and know what it can and cannot do for them generally love it. The benefits extend beyond optimization. Often the biggest opportunity for optimization arises when more than one department of an organization is involved, for instance when decisions taken by the forestry, logging or sales departments affect each other. To set up a linear programming model requires that a lot of assumptions be made. Each department must satisfy itself that the assumptions about its own operations are correct. Discussions about how to operate can then be centred on a plan based on a common set of assumptions. Communication between departments is then improved. Also, a linear programming model can be used retroactively, to show *how* events or departures from plan that happened in a period under review affected the overall profitability.

An objection often raised is that the quality of the raw material, and the prices that can be obtained for products is too uncertain to justify quantitative techniques. Although there may be some validity to this, by using linear programming to answer "what if" questions, it is possible to determine how to operate in the manner *most likely* to be the most profitable. Substantial profit improvements can then be made.

It is important that the linear programming model be applied by people familiar with the process being optimized, rather than theorists who know only the mathematics. In general it is how linear programming is applied, how human factors are handled, and the support of management, that determine how successful it will be.

Generally, a linear programming application requires computer software modules for input, display and editing of data tables, for pre-processing the input data into the form required by the linear program, including calculation of yields, etc., for generating the linear programming matrix, for finding the optimum solution, for additional processing of the results, and for generating displays and printed reports.

2. Application to a Logging, Sawmilling and Pulpmill Complex - Use of the SAWSIM® Sawmill Simulation Program

In the logging, sawmill, and pulpmill complex case, important variables are usually the volumes of raw materials of each type that are harvested in a given time period, the volumes of each type hauled to each of various locations (possibly after sorting), the quantities of merchandised logs of each type used by each processing plant, and the volumes of various lumber products and chips produced. Where there is a pulpmill, the possibility of chipping roundwood and purchasing chips from outside sources are also important variables.

The equations reflect the relationships between these variables, taking into account, for instance, the volumes and costs of each type of raw material available in the time period, the yields of merchandised logs of each type that will be generated from each forest type, the yields of lumber and chips from each type of merchandised log, the machine times required to process the pieces, the market demands and prices for products, and the capacities and operating costs of the various processing plants.

These items are input data to the model. In addition, the availability of each forest type and purchased chips at given prices, the yields of merchandised logs and products, the plant capacities, and sales demands and prices for products are also inputs to the model. They must either be known in advance, or assumptions must be made about them.

Generally speaking, forest inventory data is used to determine the number of stems in each species, diameter and height class for each forest type, and to simulate the bucking process to calculate yields of merchandised logs in each diameter and length class from stems in each diameter and height class.

A sawing simulation is the best way to generate data for lumber and chip yields, and machine time requirements in the sawmill. Reliance on data from test runs is too inflexible. However, it is important that the calculated yields be adjusted to agree with historic yield data or mill test run results.

The **SAWSIM®** Sawmill Simulation Program is an important tool here, since it can take actual shapes of logs into account, simulate any sawing pattern, and calculate machine piece counts and time requirements. Indeed the need for this type of data for linear programming applications was one of the principal reasons the program was written in the first place, and was an important factor in its design. It has been proven many times that it can calculate realistic lumber and chip yields. Routines exist for converting **SAWSIM®** results into the form required by a linear program.

SAWSIM® can simulate a sawmill when the number of logs of each type sawn on each processing line in the sawmill has been determined in advance, when there are no constraints on the volumes of individual lumber items or on the ratios in which lumber items may be produced, and when there are no limitations on machine time availabilities. There are many applications of **SAWSIM**® where these constraints are not important. When the whole operation is to be simulated, however, a combination of **SAWSIM**® and Linear Programming is the best approach, and the potential for profit improvement is often much greater than for application of the **SAWSIM**® program alone.

What Can a Linear Programming Model Show?

Generally speaking, the first step is to develop a set of results for a previous time period, say a quarter or a year, to fix the inputs and major operating variables to what they actually were, to compare the results to what was actually achieved, and to adjust the model as necessary. This is necessary for both the developers and potential users of the model to have confidence in its predictions.

The next step is to develop a "base case" for a planning period in the future which reflects the expected mode of operation. Alternative cases can then be developed, showing the effects of alternative decisions as to raw material inputs, market opportunities and possible changes to operating constraints (such as the number of operating shifts in a given facility). The output for each case is a set of values for each of the many operating variables, and the corresponding material flow and cash flow. It is desirable that the printout from the model should summarize these into a projected operating statement for the period, so that the bottom line for each case can be prepared. To keep things simple, fixed costs that do not vary from case to case should generally be ignored, or put in at standard values. Through use of the model it is usually possible to settle differences of opinion regarding the operation of the complex rather quickly, since the ultimate effect on the bottom line can be predicted.

How Easy is it to Answer Questions with the Linear Programming Model?

Of course it depends on the question, and how the model is set up. If sufficient thought was put in when the model was designed, most questions can be answered merely by changing the input data tables used by the model, and rerunning the model. This could take a few minutes to a few hours. If the alternative requires a reformulation of the model, for instance to determine the effect of a capital expenditure that will radically affect how the complex is operated, it may take days or weeks.

In addition to the values of the variables that yield the optimum solution, a linear program also calculates the marginal break-even values for intermediate products and the effects on the bottom line of small changes to the process constraints. This information can be invaluable when quick decisions have to be made and there is insufficient time to re-run the model.

It is important that good matrix generation and report writing program package be used when the model is set up. This can reduce the initial cost dramatically, and make the model much more friendly to the end user. It is important that data and results tables can be displayed and edited easily. If good software is used, changes to the structure of the model can often be made quickly.

Initially it is best for the developer of the model to run the model to answer questions. He or she then has the opportunity to check the results before showing them to the end user, and to fix any problems. Once experience has been gained, and the model has "settled down", it is best for the end user to run it.

After a period of use it is common for a model to be used less frequently. With experience it is often possible to predict what the model will show, since most changes to the data are usually relatively small. It is a good idea to keep the model up-to-date, however, since sooner or later external circumstances will force a major change in how the complex should operate, and results will be required quickly. An example in the writer's experience was the sudden and extended closure of the Suez Canal, during the Arab-Israeli War, which dramatically changed the economics of the European oil and petrochemical industries overnight. He spent the next few weeks running linear programs over and over again to assess the effects of the closure on the operations.